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The oscillatory behavior of the CoM facilitates mechanical energy balance between push-off and heel strike

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ABSTRACT

Humans use equal push-off and heel strike work during the double support phase to minimize the mechanical work done on the center of mass (CoM) during the gait. Recently, a step-to-step transition was reported to occur over a period of time greater than that of the double support phase, which brings into question whether the energetic optimality is sensitive to the definition of the step-to-step transition. To answer this question, the ground reaction forces (GRFs) of seven normal human subjects walking at four different speeds (1.1–2.4 m/s) were measured, and the push-off and heel strike work for three differently defined step-to-step transitions were computed based on the force, work, and velocity. To examine the optimality of the work and the impulse data, a hybrid theoretical-empirical analysis is presented using a dynamic walking model that allows finite time for step-to-step transitions and incorporates the effects of gravity within this period. The changes in the work and impulse were examined parametrically across a range of speeds. The results showed that the push-off work on the CoM was well balanced by the heel strike work for all three definitions of the step-to-step transition. The impulse data were well matched by the optimal impulse predictions ($R^2 > 0.7$) that minimized the mechanical work done on the CoM during the gait. The results suggest that the balance of push-off and heel strike energy is a consistent property arising from the overall gait dynamics, which implies an inherited oscillatory behavior of the CoM, possibly by spring-like leg mechanics.

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1. Introduction

The least costly gait is achieved when the push-off propulsion fully compensates for the collision loss during the double support phase (Donelan et al., 2002a; Kuo, 2002; Lewis and Ferris, 2008; Yeom and Park, 2011), during which the redirection of the center of mass (CoM) occurs. The duration of the CoM redirection can also be defined based on changes in work or velocity, and those durations are greater than that of the double support phase (Adamczyk and Kuo, 2009). This result brings into question whether the energetic optimality is sensitive to the definition of the step-to-step transition.

The theoretical analysis of energetic optimality was performed based on the simplest bipedal walking model, and the optimal push-off impulse was defined as an external input that minimizes the mechanical work done on the CoM during a gait cycle (Donelan et al., 2002a; Kuo, 2002; Ruina et al., 2005). The simplest walking model, however, assumes an instantaneous collision during the double support phase (Donelan et al., 2002a; Kuo, 2002; Kuo et al., 2005; Adamczyk et al., 2006) and neglects the effect of gravity on the CoM momentum change. Due to the simplicity of the model, only qualitative predictions of the change of the work and impulse in terms of gait parameters were presented (Donelan et al., 2002a; Kuo, 2002; Adamczyk and Kuo, 2009). Recently, the dynamic walking framework was extended to allow a finite time for step-to-step transitions and to incorporate the effects of gravity within this period (Yeom and Park, 2011). This model can quantify how consistent humans are with the mathematical relationships of the model from ground impulse to work on the CoM.

The purpose of this study was to examine whether different definitions of the step-to-step transition would affect the mechanical energy balance between push-off and heel strike during gait. The ground reaction forces (GRFs) of seven normal human subjects walking at four different speeds ranging from the self-selected speed to the maximum walking speed were measured. The push-off and heel strike work were computed from the GRFs during three differently defined step-to-step transitions based on force, work, and velocity. To examine the optimality of the work and impulse data, a hybrid theoretical-empirical analysis was performed using the recently published gravitational impulse model (Yeom and Park, 2011). The changes of work and impulse were also examined parametrically across a range of speeds.

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2. Methods

2.1. Experimental procedures

Seven healthy young subjects (mean age 22.7 ± 1.1 years) participated in the walking experiment after signing informed consent forms approved by the Institutional Review Board of KAIST prior to the test. The average height and body mass of the subjects were 1.68 ± 0.06 m and 67.7 ± 11.4 kg, respectively. The subjects walked on a 10-m-long, 1-m-wide walkway with three blocks of four different randomly ordered gait speeds (1.1–2.4 m/s). The gait speed was guided by auditory metronome cues at four different frequencies corresponding to the subject's preferred speed, his/her maximum gait speed, and two intermediate speeds. The subjects were instructed to walk with their preferred step lengths at each gait frequency. The ground reaction forces for three consecutive steps were measured using force platforms (AMTI, MA, USA) at a 200-Hz sampling frequency. To capture a full step on each platform, the locations of platforms were adjusted prior to each data collection period. Details of the experimental protocols have been reported previously (Kim and Park, 2011; Yeom and Park, 2011).

2.2. Data analysis

GRF data were 5th-order Butterworth low-pass filtered with a cutoff frequency of 30 Hz and were integrated to obtain the CoM velocity using integration constants that produced the average vertical and fore-aft CoM velocities during a step equal to zero and the average gait speed, respectively (Donelan et al., 2002b; Gard et al., 2004; Yeom and Park, 2011). The mechanical powers (N-m/s) applied by GRFs were calculated as a vector product of the external forces (N) and the CoM velocity (m/s). Three different durations (s) of the step-to-step transition were used, as suggested previously (Adamczyk and Kuo, 2009): the duration of double support defined by GRFs (τ_{force}), the duration between the onset of the positive push-off work and the end of negative collision work (τ_{work}), and the time interval between the minimum and maximum vertical CoM speeds (τ_{vel}). For simplicity, the duration defined by the peak vertical speed of the CoM in this manuscript was modified slightly from what was used in the previous research (Adamczyk and Kuo, 2009), where the extreme angles of the CoM velocity defined by both the vertical and forward components of the velocity vector were used. Because the forward velocity dominates the velocity vector at most speeds, the different definitions should give similar results. Impulses (N-s) in the sagittal plane and the mechanical work (N-m) done on the CoM by the push-off ($W_{p,exp}$), heel strike ($W_{h,exp}$), and gravity ($W_{g,exp}$) during the step-to-step transition were then calculated by integrating the GRFs and the mechanical power, respectively, over the three different time intervals of the step-to-step transition defined above. Specifically, the push-off and heel strike impulses were defined as $\mathbf{P}_{exp}=\int_\tau \mathbf{f}_{push}\ dt$ and $\mathbf{H}_{exp}=\int_\tau \mathbf{f}_{heel}dt$, respectively, where $\mathbf{f}_{push}$ and $\mathbf{f}_{heel}$ are the push-off and heel strike GRFs, respectively, in the sagittal plane and τ is the duration of the step-to-step transition. Similarly, the gravitational impulse $\mathbf{G}_{exp}$ is defined as the time integral of the body weight (N) over the transition duration τ such that $\mathbf{G}_{exp}=\int_\tau m\mathbf{g}dt$, where $\mathbf{g}$ is the gravitational acceleration (9.8 m/s^2).

2.3. Optimal push-off prediction

To examine the optimality of the work and the impulse data, a hybrid theoretical-empirical analysis was performed using a dynamic walking framework (Kuo, 2002; Ruina et al., 2005; Yeom and Park, 2011). At a steady gait, the net work done on the CoM during a single gait cycle should be zero, and thus the mechanical work done by the push-off (W_p) and heel strike (W_h) during the step-to-step transition has a complementary relationship with the GRF work during the part of the gait cycle not including the step-to-step transition, W_s, that is, $W_s+(W_p+W_h)=0$ (Kuo, 2002; Yeom and Park, 2011). Suppose the possibility of negative work of W_s following the excessive propulsion during the step-to-step transition is discounted such that $W_s \geq 0$ (Kuo, 2002). Then, the optimal push-off P_{opt} that minimizes the CoM work throughout the gait cycle is obtained when the push-off propulsion fully compensates for the collision loss as follows (Kuo, 2002; Ruina et al., 2005; Yeom and Park, 2011):

$$0=W_s|_{P_{opt}}=-(W_p+W_h)|_{P_{opt}} \tag{1}$$

To solve for the optimal push-off P_{opt} in terms of other impulse parameters, the GRF work W_p and W_h were derived in terms of push-off ($\mathbf{P}$), heel strike ($\mathbf{H}$) and gravity impulse ($\mathbf{G}$) from the previously published gravity impulse model (Yeom and Park, 2011) as follows:

$$\begin{cases} W_p=\frac{P}{2m}\{P+2mv^-\sin(\beta-\alpha)+2s_{hp}H\cos(\beta+\gamma)-2s_{gp}G\cos\beta\} \\ W_g=\frac{G}{2m}\{G+2mv^-\sin\alpha-2s_{pg}P\cos\beta-2s_{hg}H\cos\gamma\} \\ W_h=\frac{H}{2m}\{H-2mv^-\sin(\alpha+\gamma)+2s_{ph}P\cos(\beta+\gamma)-2s_{gh}G\cos\gamma\} \end{cases} \tag{2}$$

where $\mathbf{v}^-$ is the pre-transition velocity, m is the body mass, and α, β, and γ are the directions of the pre-transition velocity of the CoM, push-off impulse, and heel strike impulse, respectively (Fig. 1). The magnitude of each vector component is denoted by a non-bold letter. The impulse overlap parameter s_{ik} of impulse $\mathbf{I}_i$ with respect to $\mathbf{I}_k$ is defined as $s_{ik}\equiv\int_0^1\rho_i d\rho_k$, where $\rho_k(t)\equiv(\mathbf{I}_k(t)\cdot\mathbf{I}_k)/|\mathbf{I}_k|^2$(Ruina et al., 2005; Adamczyk and Kuo, 2009; Yeom and Park, 2011). $\mathbf{I}_k(t)$ is the partial impulse, the time integral of the force F up to time t, $\mathbf{I}_k(t)=\int_0^t Fdt$, which represents the fraction of the total impulse, $\mathbf{I}_k=\int_0^\tau Fdt$, applied over a finite duration τ, where $0\leq t\leq\tau$ (Ruina et al., 2005; Adamczyk and Kuo, 2009).

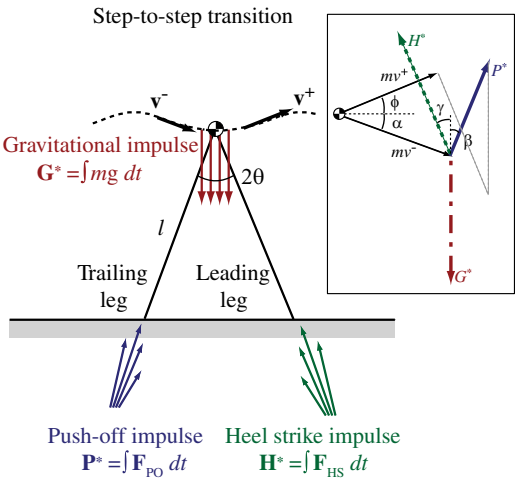

Fig. 1. Schematics of the gait parameters during a step-to-step transition. Push-off (P^*), heel strike (H^*), and gravitational impulses (G^*) generate momentum changes of the CoM from mv^- to mv^+, where v^- and v^+ are the pre- and post-step-to-step transition velocities, m is the mass of the body, and 2θ is the inter-leg angle. The directions of the pre- and post-transition velocities and the push-off and heel strike impulses are denoted as α, ϕ, β, and γ, respectively (inset).

From the momentum–impulse relationship, the heel strike impulse H is written as a function of P as follows:

$$H=(mv^-\sin(\phi+\alpha)-P\cos(\phi+\beta)+G\cos\phi)/\cos(\phi-\gamma) \tag{3}$$

where ϕ is the direction of the post-transition velocity of the CoM (Fig. 1). By combining Eqs. (2) and (3), quadratic equations of P were obtained in terms of other impulse parameters as follows:

$$a\cdot P^2+b\cdot P+c=0 \tag{4}$$

where a, b, and c are coefficients obtained from the impulse-related parameters and pre-transition velocity $\mathbf{v}$ (see the details of the equation in Appendix A). The positive solution of Eq. (4) yields the optimal push-off P_{opt}. To examine whether the measured push-off P_{exp} is correlated with other impulse measurements in an optimal manner, a, b, and c were obtained from the data, and the optimal push-off P_{opt} was calculated to compare with the P_{exp}.

To examine the change of work and impulse parametrically across a range of speeds, the impulse-related parameters used for Eqs. (2)–(4), such as the directions of the pre- (α) and post- (ϕ) transition velocities, the push-off (β) and heel strike impulses (γ) and the overlap parameters (s_{ij}) were substituted by linear regression functions in terms of the pre-transition speed v^- (Table 1). The linear function was used to describe the speed-dependent trends of both direction parameters (α,ϕ,β,γ) and overlap parameters (s_{ij}) simply without further knowledge of the relationships between those parameters and the gait speed. A model prediction for the change of the optimal impulse as a function of gait speed was obtained by solving the quadratic equations of the form of Eq. (4) with the coefficients as a function of gait speed (see Appendix A for details). The values thus predicted through the analytical model were then compared with the data. All of the above equations were derived in terms of the pre-transition speed (v^-), which has been reported to be an extremely good predictor of the gait speed v, i.e., the average forward speed (Adamczyk and Kuo, 2009); both terms are used interchangeably throughout this manuscript. For inter-subject comparison, the force (N) and mechanical work (N-m) were normalized to the body weight (N) of the subject and to the body weight (N) multiplied by the leg length (m). The significance of all of the reported trends from the parameter study was tested using regression analysis with a significance level of $p<0.05$.

3. Results

The duration of the step transition decreased with gait speed for all three definitions of the step-to-step transition (Fig. 2). Both linear and reciprocal regression of the step-to-step transition as a function of gait speed showed similar fitting performances; the

Table 1
Regression coefficients calculated for the three different step-to-step transitions and the results of the statistical tests.

Regression equation (figure number in parenthesis)		Definition of step-to-step	Slope A	Offset B	R^2	p
τ (s)$=A/v+B$	(2)	Force	0.1892	0.0051	0.7929	0.0000
		Work	0.0705	0.1431	0.0506	0.0397
		Velocity	0.1737	0.1391	0.2831	0.0000
τ (s)$=Av+B$	(2)	Force	−0.0663	0.2320	0.8073	0.0000
		Work	−0.0258	0.2294	0.0481	0.0451
		Velocity	−0.0703	0.3633	0.3091	0.0000
α (deg)$=Av+B$	(3A)	Force	0.3669	5.5269	0.0020	0.6850
		Work	1.4834	5.0115	0.0332	0.0972
		Velocity	−0.1740	9.0257	0.0004	0.8631
β (deg)$=Av+B$	(3B)	Force	6.5107	5.0540	0.5158	0.0000
		Work	3.3816	7.8005	0.1399	0.0005
		Velocity	5.3201	2.9553	0.3661	0.0000
γ (deg)$=Av+B$	(3C)	Force	0.0563	9.9264	0.0001	0.9395
		Work	0.6493	8.7609	0.0120	0.3209
		Velocity	−0.1067	9.2968	0.0006	0.8296
ϕ (deg)$=Av+B$	(3D)	Force	−4.9507	13.0938	0.4368	0.0000
		Work	−0.5637	8.5412	0.0128	0.3050
		Velocity	0.8226	7.8490	0.0085	0.4046
$s_{hp}=Av+B$	(4A)	Force	−0.0061	0.1159	0.0038	0.5791
		Work	−0.0515	0.1350	0.1875	0.0000
		Velocity	−0.0205	0.0624	0.1273	0.0009
$s_{gp}=Av+B$	(4B)	Force	−0.0174	0.3172	0.0706	0.0146
		Work	−0.0365	0.3311	0.1871	0.0000
		Velocity	−0.0895	0.4130	0.5424	0.0000
$s_{hg}=Av+B$	(4C)	Force	−0.0032	0.3402	0.0014	0.7384
		Work	−0.0261	0.3343	0.0501	0.0407
		Velocity	0.0682	0.1698	0.3781	0.0000
Data vs. model prediction (figure number in parenthesis)						
$W_{p,exp}=W_{p,model}$	(5A)	Force	–	–	0.9087	–
		Work			0.9000	
		Velocity			0.7138	
$W_{h,exp}=W_{h,model}$	(5A)	Force	–	–	0.6284	–
		Work			0.2519	
		Velocity			−1.4263	
$P_{exp}=P_{model}$	(5B)	Force	–	–	0.7203	–
		Work			0.7093	
		Velocity			0.9190	
$H_{exp}=H_{model}$	(5C)	Force	–	–	0.7364	–
		Work			0.8244	
		Velocity			0.9301	
Model prediction of work and impulse as a function of gait speed (figure number in parenthesis)						
$W_{p,h,exp}$ vs. v^- and $W_{p,h}(v^-)$ vs. v^-	(6A)	Force	–	–	−0.116 (push-off)	–
		Force			0.2275 (heel strike)	
$I_{p,h,exp}$ vs. v^- and $I_{p,h}(v^-)$ vs. v^-	(6B)	Force	–	–	0.3954 (push-off)	–
		Force			0.3010 (heel strike)	
$W_{p,h,exp}$ vs. v^- and $W_{p,h}(v^-)$ vs. v^-	(6C)	Work	–	–	−0.1527 (push-off)	–
		Work			0.4020 (heel strike)	
$I_{p,h,exp}$ vs. v^- and $I_{p,h}(v^-)$ vs. v^-	(6D)	Work	–	–	−0.0503 (heel strike)	–
		Work			−0.0517 (push-off)	
$W_{p,h,exp}$ vs. v^- and $W_{p,h}(v^-)$ vs. v^-	(6E)	Velocity	–	–	0.1010 (push-off)	–
		Velocity			0.4250 (heel strike)	
$I_{p,h,exp}$ vs. v^- and $I_{p,h}(v^-)$ vs. v^-	(6F)	Velocity	–	–	0.4751 (push-off)	–
		Velocity			0.0788 (heel strike)	

best fit was obtained for the double support data ($R^2 \sim 0.8$, Table 1). Most of the direction parameters, such as α, ϕ, and γ, did not change significantly with speed, whereas the direction of the push-off impulses (β) consistently increased with speed ($p < 0.05$) (Fig. 3, Table 1). The longest duration of the step-to-step transition defined by the CoM velocity led to a greater ratio of push-off precedence over the heel strike and yielded the smallest overlap parameters s_{hp} among three differently defined durations (Fig. 4).

Before an optimal push-off was found, it was examined whether the COM work data were well parameterized in terms of the impulse-related parameters as derived in Eq. (2). The GRF work data were well approximated by the model predictions (Fig. 5A). Specifically, the model predicted push-off work well ($R^2 > 0.7$) but predicted more heel strike collision work than was observed ($R^2 < 0.7$) (Fig. 5A, Table 1). Heel strike loss data for the duration based on the GRF were slightly better matched with the model prediction compared to the data for the other durations (Fig. 5A). The optimal push-off impulse was predicted from the mechanical energy balance equation (Eq. (1)) with the GRF work expressed in terms of impulse-related parameters. For all three definitions of the duration of the step-to-step transition, the impulse data agreed well with the optimal push-off and heel strike predictions ($R^2 > 0.7$), which were obtained by minimizing the CoM mechanical work during the gait cycle (Eqs. (3) and (4), Fig. 5B, C).

The velocity dependence of the impulses and mechanical work was quantitatively predicted by substituting the impulse-related

parameters in Eqs. (2)–(4) with linear functions of gait speed, as shown in Table 1 (Fig. 6). The model was able to predict the overall trends of the mechanical work and impulses as a function of the gait speed regardless of the large deviation of the data from the model prediction, which arose from the deviations of the impulse parameters from the regression fit (Figs. 3, 4, and 6, Table 1). The GRF work increased with speed, and most of the GRF impulses slightly decreased with gait speed (Fig. 6), whereas the speed proportional increase of the heel strike impulse defined over the duration from CoM velocity was observed (Fig. 6F).

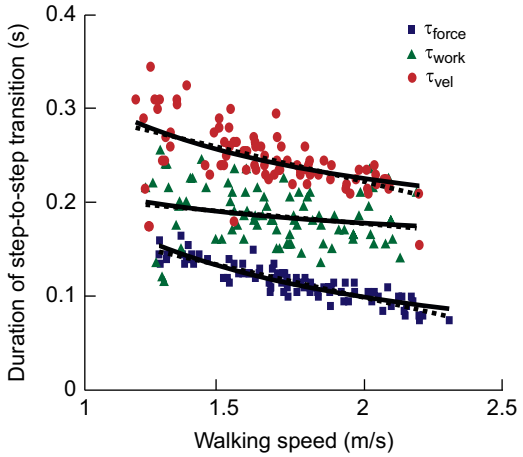

Fig. 2. The durations of the step-to-step transition as a function of walking speed defined by the double support phase (τ_{force}, blue square ■), the zero crossings of mechanical power (τ_{work}, green triangle, ▲), and the change in the peak vertical CoM velocities (τ_{vel}, red circle, ●). The same color schemes are used consistently throughout the manuscript. Data were regressed with both linear (dashed) and reciprocal (solid) functions of gait speed. Full equations and fitting parameters are listed in Table 1. Walking speed is approximated by pre-transition speed (v^-) in all figures and regressions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4. Discussion

We observed the mechanical energy balance between the push-off propulsion and heel strike loss during the three differently defined durations of the step-to-step transition (Fig. 6A, C, E). The impulse data showed a reasonably good match with the model predictions for optimal impulse for all three definitions of the step-to-step transition (Fig. 5B and C). The observed robustness of energetic optimality may be attributable to the seemingly symmetric oscillatory behavior of the CoM during the step-to-step transition. Recent studies have described the motion of the CoM of the body during the gait cycle as the oscillation of the inertia on a compliant leg (Blickhan, 1989; Geyer et al., 2006; Whittington and Thelen, 2009; Kim and Park, 2011). Due to the intrinsic symmetry of the mechanical power of an oscillatory mechanism around the minimum and maximum heights of the CoM (Fig. 7), the mechanical powers of the mass before and after the midpoint of the double support phase are approximated to be the same in magnitude but opposite in sign, showing energetic symmetry (Fig. 7). Therefore, for the definitions of the step-to-step transition that extend the duration of the double support phase in a symmetrical manner, the sum of the mechanical power of the CoM during pre-transition and post-transition should be approximately zero, which leads to the mechanical energy balance between the push-off and the heel strike.

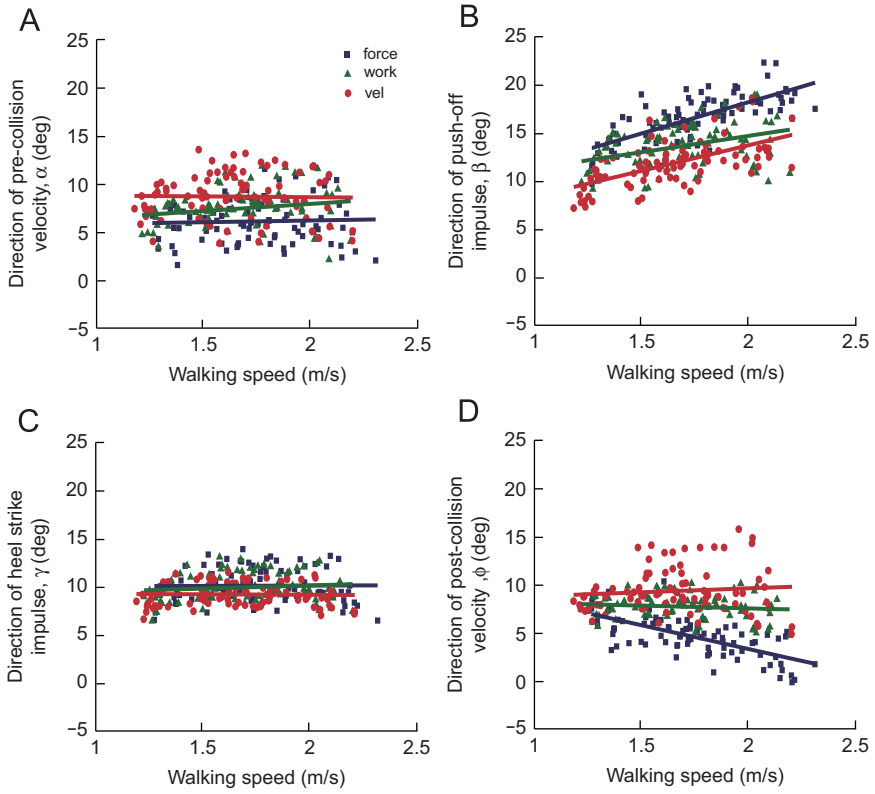

Fig. 3. The direction of the (A) pre- and (D) post-transition velocities and (B) push-off and (C) heel strike impulses. The data and gait parameters calculated for the durations based on GRF, power, and velocity are marked with blue squares (■), green triangles (▲), and red circles (●), respectively, consistently throughout the manuscript. The solid lines indicate the linear regression results as a function of walking speed. Full equations and fitting parameters are listed in Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

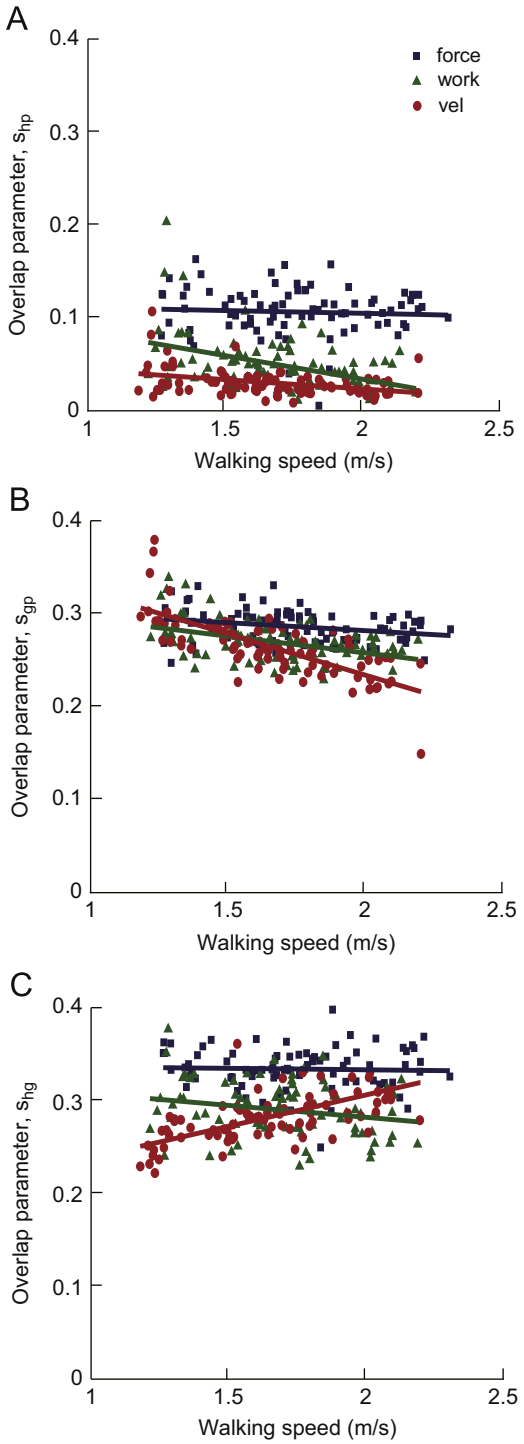

Fig. 4. Overlap parameters of (A) s_{hp}, (B) s_{gp}, and (C) s_{hg}. The impulse overlap parameter s_{ik} of impulse $\mathbf{I}_i$ with respect to $\mathbf{I}_k$ is defined as $s_{ik} \equiv \int_0^1 \rho_i d\rho_k$, where $\rho_k(t) \equiv (\boldsymbol{I}_k(t) \cdot \boldsymbol{I}_k)/|\boldsymbol{I}_k|^2$. $\mathbf{I}_k(t)$ is the partial impulse, the time integral of the force F up to time t, $\boldsymbol{I}_k(t) = \int_0^t F dt$, which represents the fraction of the total impulse, $\boldsymbol{I}_k = \int_0^\tau F dt$, applied over a finite duration τ, where $0 \le t \le \tau$. The solid lines indicate the linear regression of overlap parameters as a function of walking speed. Full equations and fitting parameters are listed in Table 1.

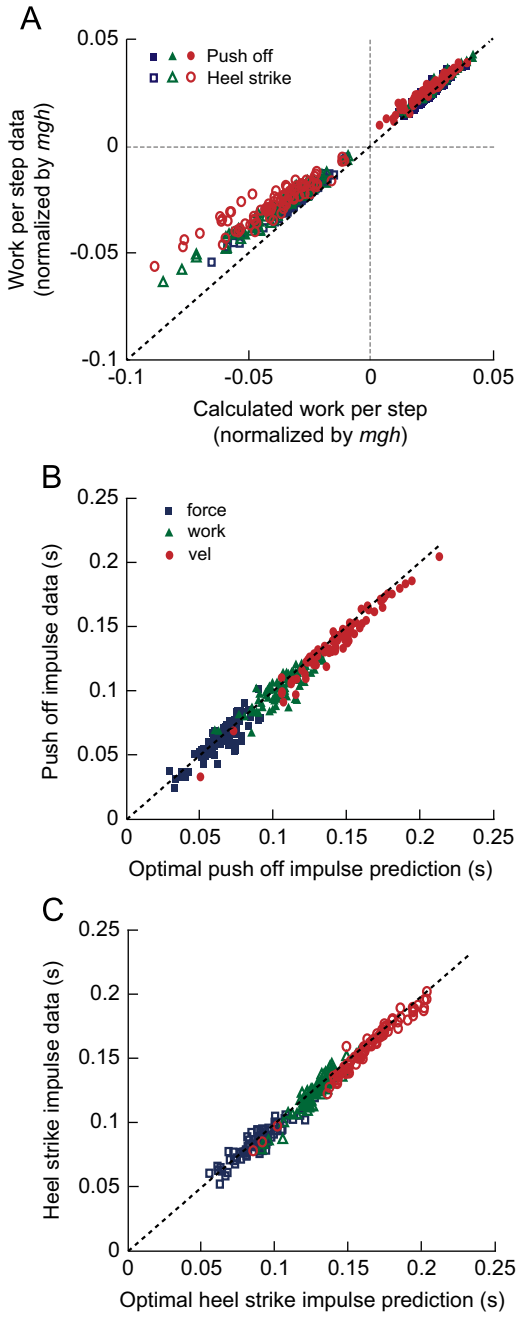

Fig. 5. Empirical data and model predictions of the impulse and mechanical work for the three definitions of the step-to-step transitions. The color schemes follow the previous notations. The filled (■, ▲, ●) and hollow (□, △, ○) markers indicate the parameters for push-off and heel strike, respectively. (A) GRF work data versus the model predictions for work with the impulse-related parameters for Eq. (2) obtained empirically. (B) Push-off impulse data versus the predicted optimal push-off that was computed from equations that minimize the CoM work in the model (Eqs. (2)–(4), also see Appendix A). (C) Heel strike impulse data versus the predicted optimal heel strike. Work and impulses were normalized by mgh and mg, respectively, where m, g, and h are the body mass of the subject, the gravitational acceleration, and the leg length of the subject, respectively. Dotted lines indicate the equality lines for the x and y axes. Goodness of fit data are listed in Table 1.

The decreasing trends of the duration of the step-to-step transition defined by GRF, CoM work, and CoM velocity were consistent with those of previous reports (Kirtley et al., 1985; Adamczyk and Kuo, 2009; Dubbeldam et al., 2010). The regression fit of the reduced step duration data using both linear and nonlinear, i.e., reciprocal, functions of gait speed showed similar fitting performance ($\Delta R^2 < 10\%$, Table 1), as reported previously (Yeom and Park, 2011). Compared to the GRF-based definition, the durations defined by the power and velocity showed greater variability, and the degree of variability became greater at slower gait speeds (Fig. 2). The observed variability may be attributed to

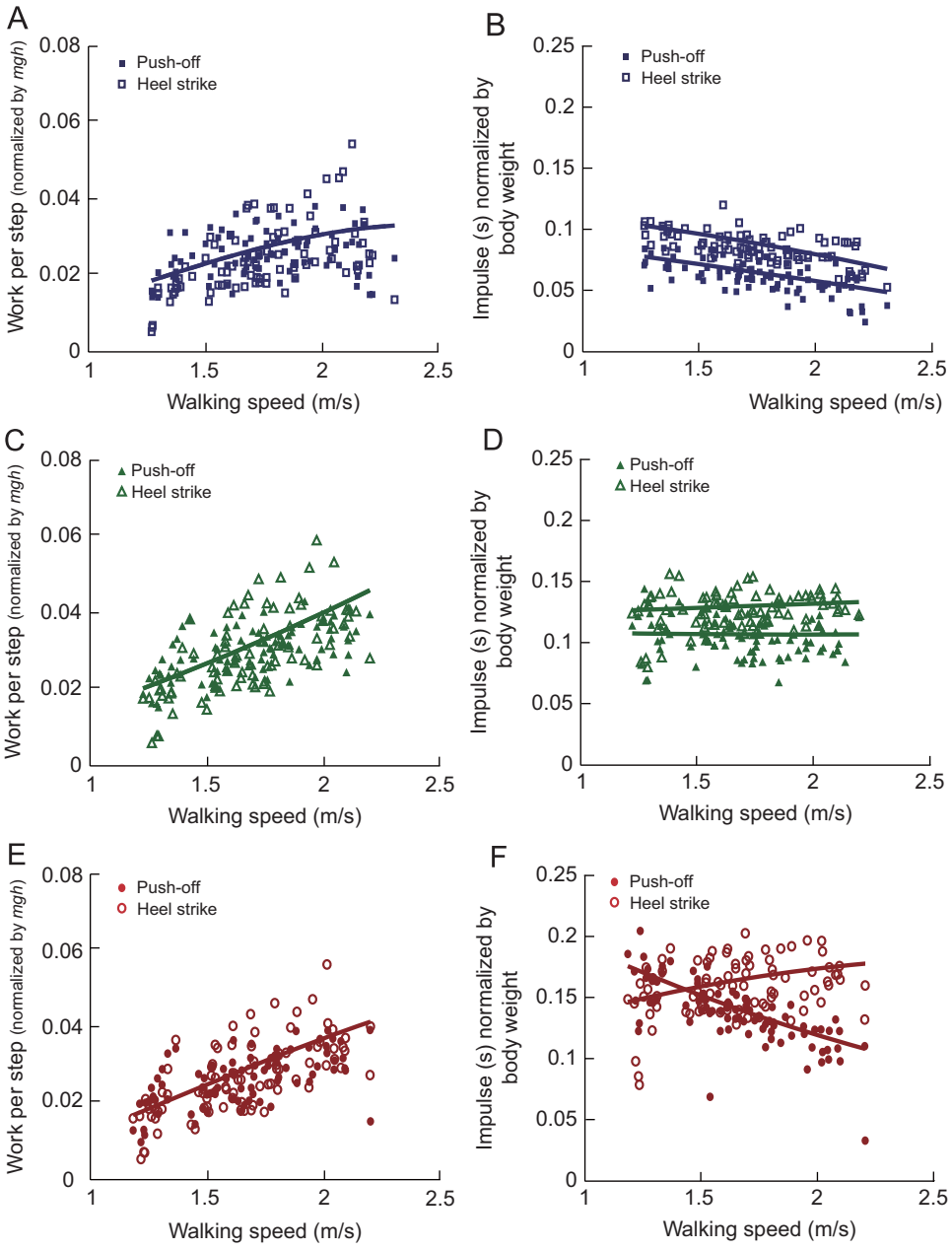

Fig. 6. Data and model predictions for work and impulse as a function of gait speed. The color scheme follows that used in previous figures, and the filled (■, ▲, ●) and hollow (□, △, ○) markers indicate the parameters for push-off and heel strike, respectively. Mechanical work and impulse during the step-to-step transition defined by (A, B) double support, (C, D) power, and (E, F) CoM velocities. Solid lines show model predictions of optimal work and impulse as a function of gait speed, which were obtained from the parameterization of the impulse-related model parameters in Eqs. (2)–(4) as linear regression functions of the gait speed (Eq. (A2) and Table 1). For optimal gait, push-off work balances the heel strike work, such that the work plots (A, C, E) have only one overlapped prediction curve. Fitting results are listed in Table 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the error associated with the estimation of the CoM velocity, which could be generated while assigning integration constants or while eliminating drift noise (Donelan et al., 2002a; Gard et al., 2004). The drift noise of the force plate data may have caused the error in the estimation of CoM velocity while integrating GRFs, and, thus, the smaller signal (GRFs) to noise (drift) ratio would produce greater variability at slower speeds. However, the direction of the heel strike impulse (γ) did not change significantly, and the direction of the push-off impulse (β), i.e., the ratio of the forward to vertical impulses, increased with gait speed (Fig. 3B), which may be required to propel the CoM forward with more force at faster gait speeds given the limited plantar flexor torque (Fugl-Meyer, 1981; Cunningham et al., 1987; Zajac, 1989; Neptune and Sasaki, 2005) as opposed to the proportional increase of the heel strike loss with the speed. The average data of overlap parameters had the same order of magnitude as those previously published (Adamczyk and Kuo, 2009; Yeom and Park, 2011) but were smaller in magnitude due to the different gait speed ranges for averaging. Compared to the data previously published by Adamczyk and Kuo (2009), the results here showed greater variability, i.e., smaller R^2, requiring the improved consistency of the collected data.

The model predictions for CoM work matched the push-off work data, $W_{p,exp}$, reasonably well ($R^2 > 0.7$) but showed greater

deviation for heel strike loss $W_{h,exp}$ ($R^2 < 0.7$) (Fig. 5A). The errors were greater for the data estimated over the duration defined with the CoM velocity. The results imply that the possible accumulation of drift artifacts that were not fully corrected for during the estimation of the CoM velocity may have resulted in erroneous heel strike impulse estimates that, in turn, induced errors in the heel strike loss estimates. Despite the deviations of the $W_{h,exp}$ data from the model prediction, the impulse data matched reasonably well with the optimal impulse predictions ($R^2 > 0.7$, Fig. 5B and C), suggesting that the balancing of the heel-strike and push-off energy is a consistent property arising from the overall gait dynamics.

The observed increase in the push-off work with gait speed is required to generate greater propulsion to compensate for the greater loss at faster gait speeds (Fig. 6A, C, and E). By the definition of an impulse, i.e., the time integral of the GRF, the magnitude and the change of an impulse are affected by changes in the duration and in the GRF. It was observed that the GRF magnitudes did not change significantly, and thus the observed decrease in impulse with higher speeds (Fig. 6B, D, and F) was caused mainly by decreasing the duration (Fig. 2), except for the heel strike impulse data defined over the duration of the CoM velocity change (Fig. 6F). The seemingly conflicting trend of the heel strike, i.e., the increase with speed, was due to the inclusion of a sharp increase in the heel strike GRF during the integration of the force over an extended duration of the step-to-step transition, whereas large peaks in the heel strike force were mostly or partly excluded for the integration of the GRF over the other two definitions of duration (Fig. 6F). The inclusion of large peaks of heel strike GRF during the force integration to obtain impulse was reflected in the increased overlap parameters of s_{hg} (Fig. 4C), which were then implemented in the model predictions.

The large deviation of the data from the model prediction arose from the deviations of the impulse parameters from the regression fit (Figs. 3, 4, and 6). Compared to young subjects, greater variability is expected in the data for elderly patients or those with conditions affecting gait. Therefore, ways to reduce the variability, such as data collection on a treadmill (Donelan et al., 2002b; Dingwell and Kang, 2007; Yeom and Park, 2011), should be explored to increase the consistency between the data and the model predictions. Finally, due to the lack of a gait model that predicts the change of gait parameters with speed, the model predictions for work and impulse were calculated based on the empirical model parameters of α, β, γ, s_{hp}, s_{gp}, and s_{hg}; thus, the model predictions presented in this study were descriptive rather than predictive.

Conflict of interest

None

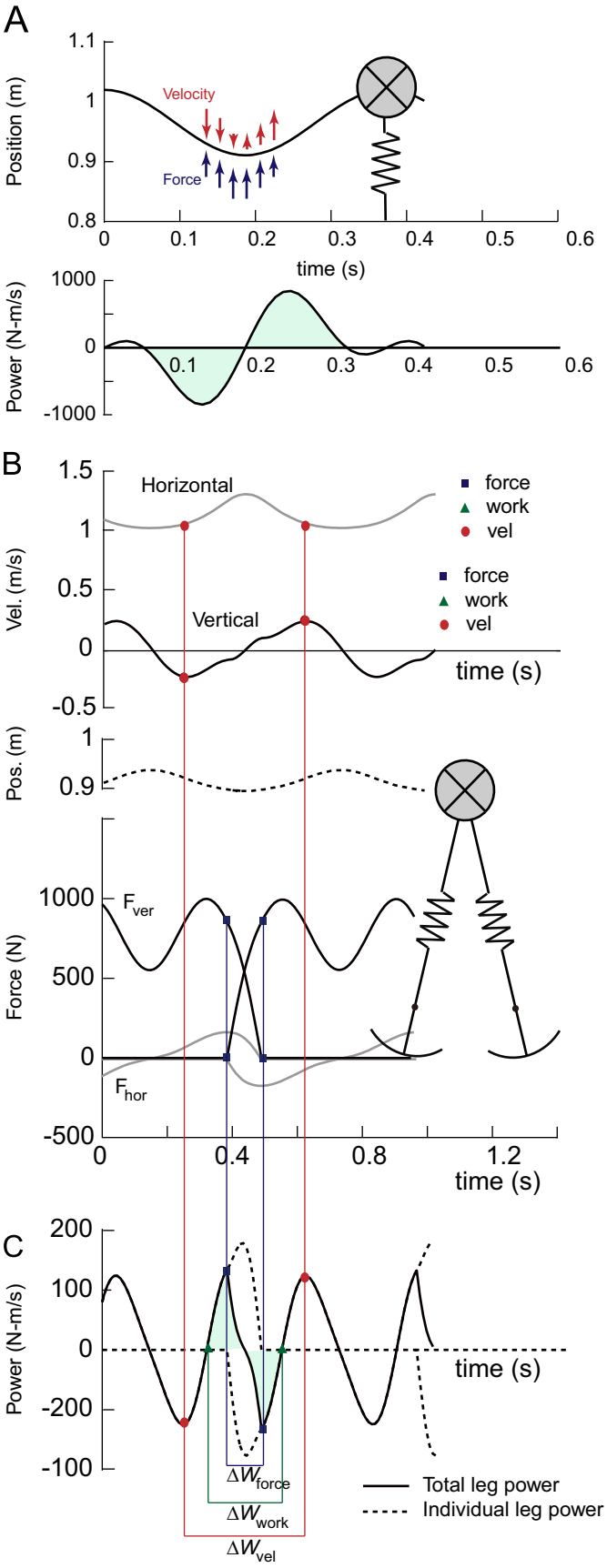

Fig. 7. The dynamics and power of a simple mass-spring system. (A) Position (upper panel) and mechanical power (lower panel) of a vertically oscillating mass-spring system. Before and after the minimum height of the CoM, the direction of the CoM velocity (red) changes, whereas the direction of the ground reaction force (blue) does not change. The mechanical power of the mass before and after the minimum height are thus the same in magnitude but opposite in sign, giving a net sum of the mechanical energy around the minimum height of CoM to be approximately zero. (B) Dynamics and (C) power of a bipedal walking model simulation with compliant legs. With respect to the minimum height of the CoM position, which is approximated to occur at the middle of the double support period, and the mechanical power before and after appears to be the same in magnitude and the opposite in sign. Therefore, as long as the definition of the duration of the step-to-step transition is in the form of a temporally symmetric extension of the duration of the double support phase, the push-off propulsion energy should be approximately balanced by the heel strike loss. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Appendix A

Calculation of optimal push-off

The least costly gait is achieved when the push-off propulsion fully compensates for the collision loss (Kuo, 2002; Yeom and Park, 2011), that is, when $W_p+W_h=0$, where W_p and W_h are obtained from Eq. (2). Substituting the heel strike impulse H expressed in terms of the push-off impulse P and other gait parameters, as shown in Eq. (3), into the mechanical energy balance equation, the mechanical energy equation is obtained in the form of a quadratic equation of push-off P, as follows:

$$0=W_p+W_h=a\cdot P^2+b\cdot P+c$$

$$\text{where}\begin{cases} a=\cos^2(\phi-\gamma)-2s_{hp}\cos(\beta+\gamma)\cos(\phi+\beta)\cos(\phi-\gamma) \\ +\cos^2(\phi+\beta)-2s_{ph}\cos(\beta+\alpha)\cos(\phi+\beta)\cos(\phi-\gamma) \\ b=mv^-\sin(\beta-\alpha)\cos^2(\phi-\gamma)+mv^-\cdot s_{hp}\sin(\phi+\alpha)\cos(\beta+\gamma)\cos(\phi-\gamma) \\ +s_{hp}G\cos(\beta+\gamma)\cos(\phi-\gamma)\cos\phi-s_{gp}G\cos^2(\phi-\gamma)\cos\beta \\ -mv^-\sin(\phi+\alpha)\cos(\phi+\beta)-G\cos(\phi+\beta)\cos\phi \\ +mv^-\sin(\alpha+\gamma)\cos(\phi+\beta)\cos(\phi-\gamma) \\ +mv^-\cdot s_{ph}\sin(\phi+\alpha)\cos(\beta+\gamma)\cos(\phi-\gamma) \\ +s_{ph}G\cos(\beta+\gamma)\cos(\phi-\gamma)\cos\phi \\ +s_{gh}G\cos(\phi+\beta)\cos(\phi-\gamma)\cos\gamma \\ c=m^2v^{-2}\sin^2(\phi+\alpha)+G^2\cos^2\phi+2mv\,G\sin(\phi+\alpha)\cos\phi \\ -2m^2v^{-2}\sin(\alpha+\gamma)\sin(\phi+\alpha)\cos(\phi-\gamma) \\ -2mv^-G\sin(\alpha+\gamma)\cos(\phi-\gamma)\cos\phi \\ -2mv^-\cdot s_{gh}G\sin(\phi+\alpha)\cos(\phi-\gamma)\cos\gamma \\ -2s_{gh}G^2\cos(\phi-\gamma)\cos\phi\cos\gamma \end{cases} \quad \text{(A1)}$$

The positive solution of the above equation obtained using the quadratic formula is the optimal push-off P_{opt}: $P_{opt}=(-b+\sqrt{b^2-4ac})/2a$. The impulse-related parameters (β, γ, s_{hp}, s_{gp}, s_{hg}, and G in Eq. (A1)) and the pre-transition velocity $\mathbf{v}^-$ were obtained empirically.

Changes in the work and impulse as a function of walking speed

To parametrically examine the change of work and impulse across a range of gait speeds, the impulse parameters in Eq. (A1), which were empirically obtained in the previous section, were substituted by linear functions of the walking speed (Table 1). The equations of CoM work during the step-to-step transition were then obtained as a quadratic function of P with coefficients that are functions of the gait speed, as follows:

$$0=W_p+W_h=a(v^-)\cdot P^2+b(v^-)\cdot P+c(v^-) \quad \text{(A2)}$$

where $a(v^-)$, $b(v^-)$, and $c(v^-)$ are obtained by combining Eq. (A1) and the regression equations in Table 1.

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